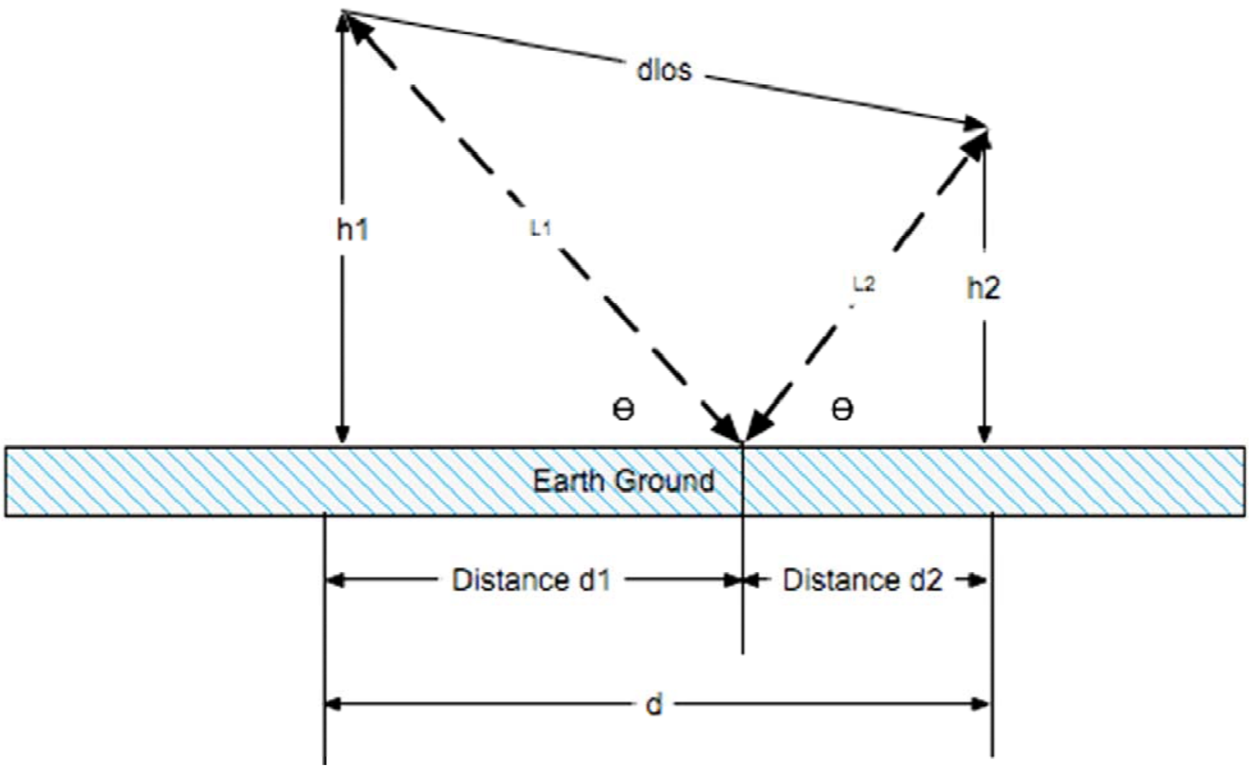
The Problem



Two antennas of height h1 and h2 placed a distance d apart has a direct path and an indirect path resulting in a multipath situation. The direct path will obey the standard path loss equation, but the indirect path will arrive at a later time than the direct path causing interference with the direct path signal resulting in nulls and fades. This paper will develop the mathematics necessary to deal with this situation.

To begin we will derive the equation for ground bounce multipath by computing the distances for L1, L2 as a function of h1, h2, and d

From physics, the angle of incidence must equal the angle of reflection. Thus the two angles Θ are equal. This means that the cos, sine and tangents of the two triangles must be equal.

To begin we note the following equations

$$\sqrt{(\mathrm{h1})^2+\mathrm{d1}^2}=\mathrm{L1},;\sqrt{(\mathrm{h2})^2+\mathrm{d2}^2}=\mathrm{L2};\mathrm{dlos}=\sqrt{(\mathrm{h1}-\mathrm{h2})^2+d^2}$$

$$\mathrm{Cos}[\theta]=\frac{\mathrm{d1}}{\mathrm{L1}}=\frac{\mathrm{d2}}{\mathrm{L2}},\mathrm{Tan}[\theta]=\frac{\mathrm{h1}}{\mathrm{d1}}=\frac{\mathrm{h2}}{\mathrm{d2}},\mathrm{Sin}[\theta]=\frac{\mathrm{h1}}{\mathrm{L1}}=\frac{\mathrm{h2}}{\mathrm{L2}}$$

Or

$$\frac{\mathrm{d1}}{\mathrm{L1}} = \frac{\mathrm{d2}}{\mathrm{L2}}; \frac{\mathrm{h1}}{\mathrm{d1}} = \frac{\mathrm{h2}}{\mathrm{d2}}; \frac{\mathrm{h1}}{\mathrm{L1}} = \frac{\mathrm{h2}}{\mathrm{L2}}$$

Finally, we know

$$d = \mathrm{d1} + \mathrm{d2}$$

We can now use these equations to solve for the following

$$\mathrm{L1} = \mathrm{h1}\sqrt{1 + \frac{d^2}{(\mathrm{h1} + \mathrm{h2})^2}}$$

$$\mathrm{L2} = \mathrm{h2}\sqrt{1 + \frac{d^2}{(\mathrm{h1} + \mathrm{h2})^2}}$$

$$\mathrm{d1} = \frac{d\mathrm{h1}}{\mathrm{h1} + \mathrm{h2}}$$

$$\mathrm{d2} = \frac{d\mathrm{h2}}{\mathrm{h1} + \mathrm{h2}}$$

$$\Theta = \mathrm{ArcTan}[\frac{\mathrm{h1}}{\mathrm{d1}}] = \mathrm{ArcTan}[\frac{\mathrm{h1}}{\frac{d\mathrm{h1}}{\mathrm{h1} + \mathrm{h2}}}] = \mathrm{ArcTan}[\frac{\mathrm{h1} + \mathrm{h2}}{d}]$$

The multipath distance, mp is:

$$\mathrm{mp} = \mathrm{L1} + \mathrm{L2} = \sqrt{d^2 + (\mathrm{h1} + \mathrm{h2})^2}$$

And the direct line of sight distance, dlos can be shown to be:

$$\mathrm{dlos} = \sqrt{(\mathrm{h1} - \mathrm{h2})^2 + d^2}$$

Now we use these distances to create time delays that can be converted to angles. The assumption will be that frequency is in MHz.

When the difference in the phase angle from the direct path to the multipath is equal to odd multiples of π, the direct path and multipath signals will cancel creating a null.

$$\mathrm{tm} = \frac{\mathrm{mp}}{c} = \frac{\mathrm{mp}}{300}$$

$$\mathrm{td} = \frac{\mathrm{dlos}}{c} = \frac{\mathrm{dlos}}{300}$$

Where tm is the time delay for indirect path signal and td is the direct path time delay.

$$\text{adp} = 2\pi f \text{td}$$

$$\text{adm} = 2\pi f \text{tm}$$

$$\text{AngDiff} = \text{adm} - \text{adp}$$

Where adp is the phase angle of the direct path at the receiving antenna, adm is the phase angle of the indirect path signal at the receiving antenna and AngDiff is the difference between the two.

Substituting and simplifying, we obtain the following for AngDiff:

$$\text{AngDiff} = \frac{2\pi f\,(-\sqrt{d^2 + (\text{h1} - \text{h2})^2} + \sqrt{((\text{h1} + \text{h2})^2 + d^2}))}{c}$$

Where c is the speed of light

For f in MHz

$$\text{AngDiff} = \frac{2\pi f\,(-\sqrt{d^2 + (\text{h1} - \text{h2})^2} + \sqrt{((\text{h1} + \text{h2})^2 + d^2}))}{300}$$

The Reflection Coefficient r

The amount of the reflected signal is determined not only by the distances between the two antennas, but by the amount of energy that is reflected by the ground in the indirect path. This is determined by the reflection coefficient, r of the ground. Much of the following comes from the paper "Radio Propagation at frequencies above 30 Megacycles" by Kennth Bullington published in Proceedings of the IRE--Waves and Electrons Section Oct 1947. Unfortunately, Bullington made two simplifying assumptions that create considerable error at short range microwave frequencies common in WiFi, and other wireless applications in use today. The first of these assumptions involves AngDiff:

Bullington's first assumption is that the AngDiff is equal to

$$\text{BullingtonsAngDiff} = \frac{4\pi f \text{h1h2}}{cd} = \frac{4\pi f \text{h1h2}}{300d} = \frac{\pi f \text{h1h2}}{75d}$$

If one does a series expansion of AngDiff and takes the first term, one will obtain BullingtonsAngDiff.

Now the reflection coefficient is:

$$r = \frac{\text{Sin}[\theta] - z}{\text{Sin}[\theta] + z}$$

Where z is equal to:

$$z = \sqrt{\varepsilon_0 - Cos(\Theta)^2} \text{ for Horizontal Polarization}$$

$$z = \frac{\sqrt{\varepsilon_0 - Cos(\Theta)^2}}{\varepsilon_0} \text{ for Vertical Polarization}$$

Where ϵ_0 is:

$$\varepsilon_0 = \varepsilon - j60\sigma\lambda$$

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ϵ is the Dielectric constant of ground relative to unity in free space.

σ is the conductivity of ground in Siemens/meter

λ is the wavelength

j is the imaginary operator.

Let's look at z in more detail.

Horizontal Polarization

If ϵ_0 is complex and is equal to ϵ-j60 σ λ then it can be expressed in phasor form as

$$\varepsilon_0 = \varepsilon - j60\sigma\lambda$$

Substituting for horizontal polarization, z is

$$z = \sqrt{\varepsilon - j60\sigma\lambda - Cos(\Theta)^2} \text{ for Horizontal Polarization}$$

And the reflection coefficient is:

$$r = \frac{\sin(\Theta) - \sqrt{\varepsilon - j60\sigma\lambda - Cos(\Theta)^2}}{\sin(\Theta) + \sqrt{\varepsilon - j60\sigma\lambda - Cos(\Theta)^2}} \text{ for Horizontal Polarization}$$

Using Mathematica and do a complex expand on this function we obtain:

$$\mathrm{r} = \frac{-e^{\frac{1}{2}i\mathrm{Arg}[\epsilon - 60i\lambda\sigma - \mathrm{Cos}[\theta]^2]}(\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon\mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + \mathrm{Sin}[\theta]}{e^{\frac{1}{2}i\mathrm{Arg}[\epsilon - 60i\lambda\sigma - \mathrm{Cos}[\theta]^2]}(\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon\mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + \mathrm{Sin}[\theta]}$$

taking the real part gives

$$\mathrm{r} = \frac{-\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{-60\lambda\sigma}{\epsilon - \mathrm{Cos}[\theta]^2}]](\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon\mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + \mathrm{Sin}[\theta]}{\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{-60\lambda\sigma}{\epsilon - \mathrm{Cos}[\theta]^2}]](\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon\mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + \mathrm{Sin}[\theta]}$$

If we now substitute in

$$\Theta = \mathrm{ArcTan}[\frac{\mathrm{h1} + \mathrm{h2}}{d}]$$

We obtain the general equation for the reflection coefficient (r) for horizontal polarization.

$$\frac{\mathrm{h1} + \mathrm{h2} - ((d^2(-1+\epsilon) + (\mathrm{h1}+\mathrm{h2})^2\epsilon)^2 + 3600(d^2 + (\mathrm{h1}+\mathrm{h2})^2)^2\lambda^2\sigma^2)^{1/4}\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{60\lambda\sigma}{-\frac{1}{1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}} + \epsilon}]]}{\mathrm{h1} + \mathrm{h2} + ((d^2(-1+\epsilon) + (\mathrm{h1}+\mathrm{h2})^2\epsilon)^2 + 3600(d^2 + (\mathrm{h1}+\mathrm{h2})^2)^2\lambda^2\sigma^2)^{1/4}\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{60\lambda\sigma}{-\frac{1}{1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}} + \epsilon}]]}$$

Let σ = 0

If σ=0 this simplifies to

$$\mathrm{r} = \frac{-\sqrt{(\epsilon + \mathrm{Cos}[\theta]^2)} + \mathrm{Sin}[\theta]}{\sqrt{(\epsilon + \mathrm{Cos}[\theta]^2)} + \mathrm{Sin}[\theta]}$$

or

$$r_h = \frac{h1 + h2 - \sqrt{d^2(\varepsilon - 1) + (h1 + h2)^2 \varepsilon}}{h1 + h2 + \sqrt{d^2(\varepsilon - 1) + (h1 + h2)^2 \varepsilon}} \qquad \text{Horizontal Polarization}$$

Vertical Polarization

For vertical polarization things aren't so simple. We have to divide the z term by ϵ_0. Doing this gives:

$$z = \frac{\sqrt{\varepsilon_0 - Cos(\Theta)^2}}{\varepsilon_0} \text{ for Vertical Polarization}$$

Where $\varepsilon_0 = \varepsilon - j60\sigma\lambda$

To do this we will convert to polar form

$$\varepsilon_0 = \varepsilon - j60\sigma\lambda = \sqrt{\varepsilon^2 + (60\sigma\lambda)^2}\, Cos\left[Arg(\varepsilon - j60\sigma\lambda)\right]$$

Substituting this into the equation for r and simplifying gives:

$$\mathrm{r} = \frac{-e^{\frac{1}{2} i \mathrm{Arg}[\epsilon - 60 i\lambda\sigma - \mathrm{Cos}[\theta]^2]} \epsilon (\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon \mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + (\epsilon^2 + 3600\lambda^2\sigma^2)\mathrm{Sin}[\theta]}{e^{\frac{1}{2} i \mathrm{Arg}[\epsilon - 60 i\lambda\sigma - \mathrm{Cos}[\theta]^2]} \epsilon (\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon \mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + (\epsilon^2 + 3600\lambda^2\sigma^2)\mathrm{Sin}[\theta]}$$

If we take the real part of $-e^{\frac{1}{2} i \mathrm{Arg}[\epsilon - 60 i\lambda\sigma + \mathrm{Cos}[\theta]^2]}$ and then simplify, we find that this term is:

$$-\mathrm{Cos}[\frac{1}{2}\mathrm{Arg}[\epsilon - 60 i\lambda\sigma + \mathrm{Cos}[\theta]^2]]$$

Or

$$-\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{-60\lambda\sigma}{\epsilon + \mathrm{Cos}[\theta]^2}]]$$

Substituting back into r gives

$$r = \frac{-\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{-60\lambda\sigma}{\epsilon - \mathrm{Cos}[\theta]^2}]]\epsilon(\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon\mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + (\epsilon^2 + 3600\lambda^2\sigma^2)\mathrm{Sin}[\theta]}{\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{-60\lambda\sigma}{\epsilon - \mathrm{Cos}[\theta]^2}]]\epsilon(\epsilon^2 + 3600\lambda^2\sigma^2 - 2\epsilon\mathrm{Cos}[\theta]^2 + \mathrm{Cos}[\theta]^4)^{1/4} + (\epsilon^2 + 3600\lambda^2\sigma^2)\mathrm{Sin}[\theta]}$$

Substituting

$$\Theta = \mathrm{ArcTan}[\frac{\mathrm{h1} + \mathrm{h2}}{d}]$$

We obtain the general equation for the reflection coefficient for Vertical Polarization

$$r = \frac{\frac{(\mathrm{h1}+\mathrm{h2})(\epsilon^2 + 3600\lambda^2\sigma^2)}{d\sqrt{1 + \frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}}} - \epsilon(\frac{1}{(1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2})^2} - \frac{2\epsilon}{1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}} + \epsilon^2 + 3600\lambda^2\sigma^2)^{1/4}\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{60\lambda\sigma}{-\frac{1}{1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}} + \epsilon}]]}{\frac{(\mathrm{h1}+\mathrm{h2})(\epsilon^2 + 3600\lambda^2\sigma^2)}{d\sqrt{1 + \frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}}} + \epsilon(\frac{1}{(1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2})^2} - \frac{2\epsilon}{1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}} + \epsilon^2 + 3600\lambda^2\sigma^2)^{1/4}\mathrm{Cos}[\frac{1}{2}\mathrm{ArcTan}[\frac{60\lambda\sigma}{-\frac{1}{1+\frac{(\mathrm{h1}+\mathrm{h2})^2}{d^2}} + \epsilon}]]}$$

Let σ = 0

If we take the limit of this as σ approaches 0, we obtain:

$$r = \frac{-\sqrt{\epsilon - \mathrm{Cos}[\theta]^2} + \epsilon\mathrm{Sin}[\theta]}{\sqrt{\epsilon - \mathrm{Cos}[\theta]^2} + \epsilon\mathrm{Sin}[\theta]}$$

or

$$r_v = \frac{\varepsilon(h1+h2) - \sqrt{d^2(\varepsilon-1) + (h1+h2)^2\varepsilon}}{\varepsilon(h1+h2) + \sqrt{d^2(\varepsilon-1) + (h1+h2)^2\varepsilon}} \quad \text{Vertical Polarization}$$

An example of two different grounds for ϵ and σ are

	ϵ	σ	Real Part of Reflection coefficient	Imaginary Part of Reflection coefficient
Good Ground	30	.02	30	.4
Poor Ground	4	.001	4	.0075
Sea Water	80	4	80	-30

Given these numbers, for our purposes it is fair to neglect the imaginary part of the reflection coefficient especially for grounds that are not metal. Even the Sea Water ground showed little difference when actual simulations of the final equations for received power were made. Therefore for simplification reasons, we will assume that σ = 0 for the remained of this paper.

Then

$$\varepsilon_0 = \varepsilon$$

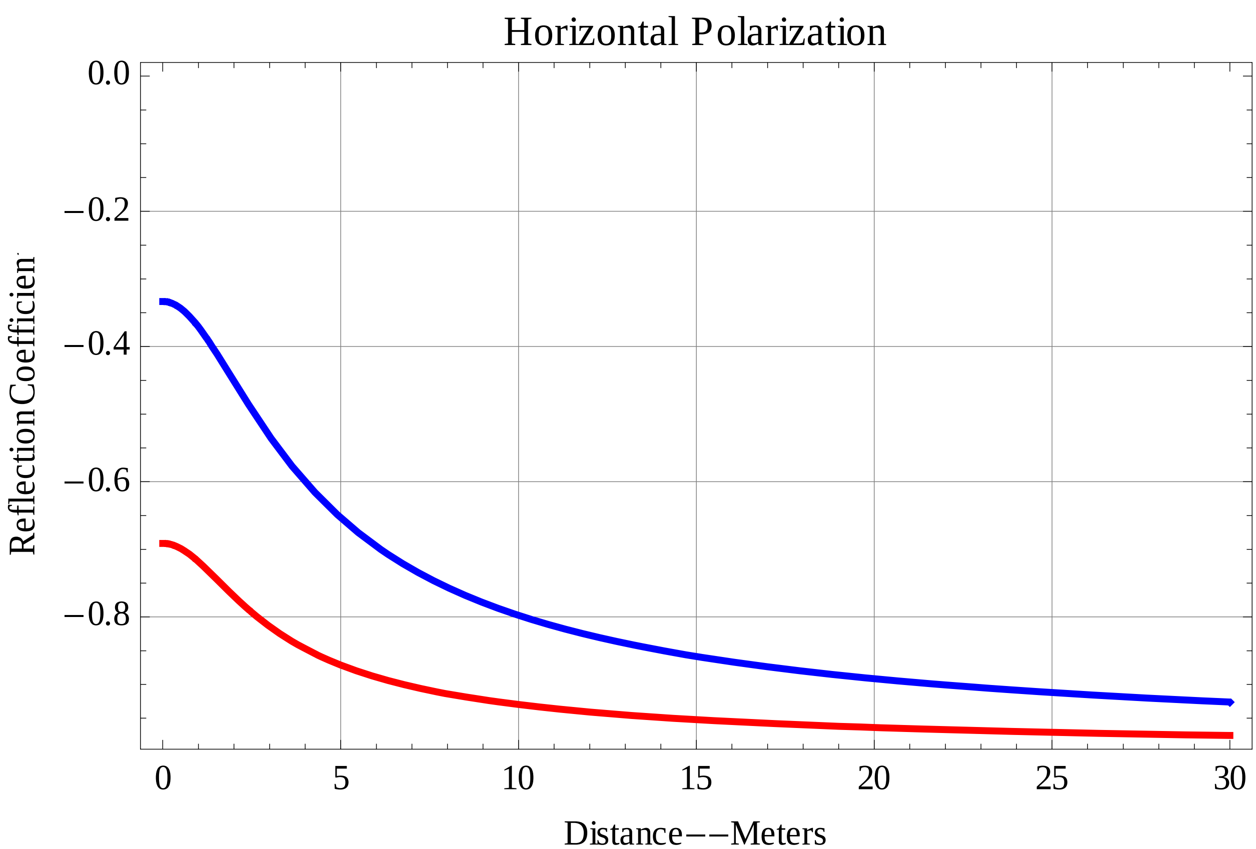
And the reflection coefficients are:

$$r_h = \frac{h1 + h2 - \sqrt{d^2(\varepsilon - 1) + (h1 + h2)^2 \varepsilon}}{h1 + h2 + \sqrt{d^2(\varepsilon - 1) + (h1 + h2)^2 \varepsilon}} \qquad \text{Horizontal Polarization}$$

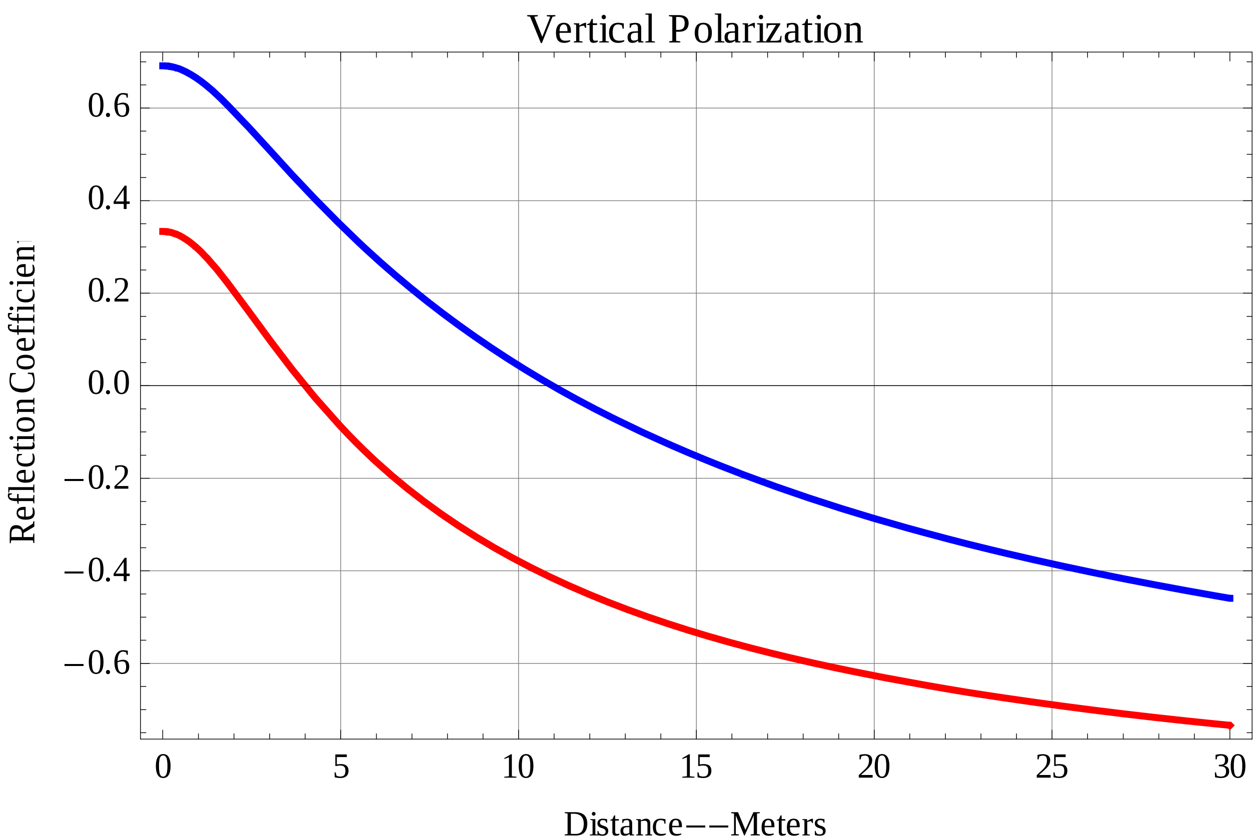
$$r_v = \frac{\varepsilon(h1 + h2) - \sqrt{d^2(\varepsilon - 1) + (h1 + h2)^2 \varepsilon}}{\varepsilon(h1 + h2) + \sqrt{d^2(\varepsilon - 1) + (h1 + h2)^2 \varepsilon}} \qquad \text{Vertical Polarization}$$

Graphs of the reflection coefficient

Let's look at a graph of r. First for horizontal polarization. The red is for ϵ=30, blue is for ϵ=4, σ =0:



Now look at Vertical Polarization. Again, the red is for ϵ=30, blue is for ϵ=4

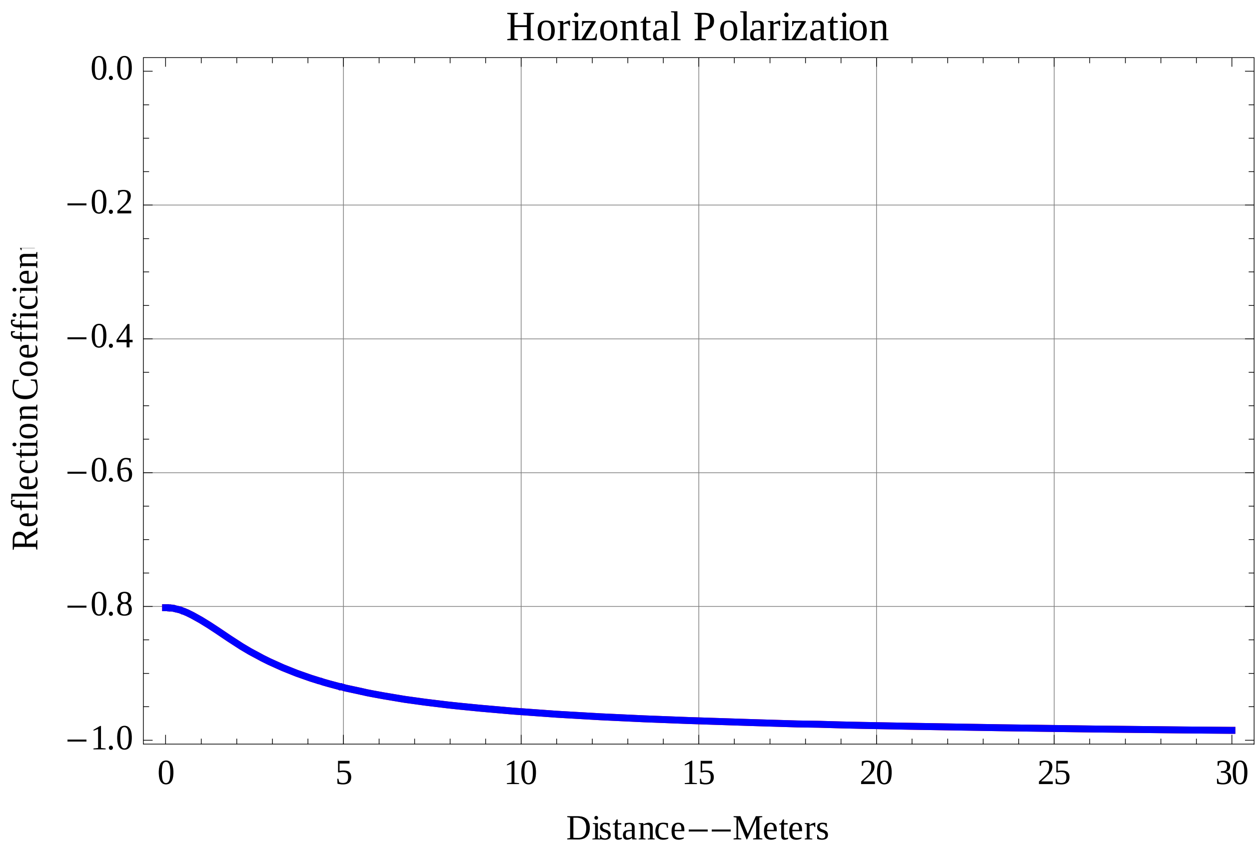


In both of these graphs, the antenna height was 1 meter for both antennas.

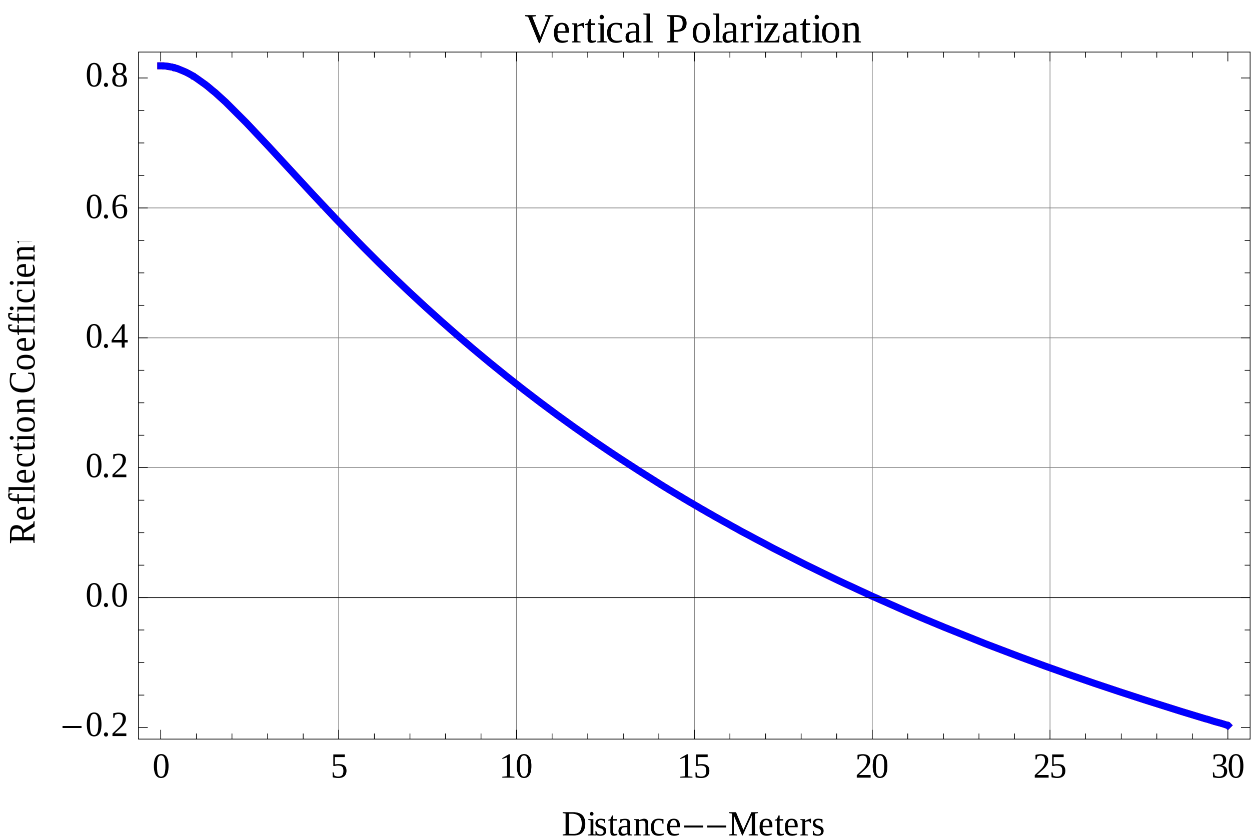
Observe that for vertical polarization, the reflection coefficient actually starts out positive, goes through zero and eventually approaches -1. The angle at which r is zero is called Brewsters angle. In both cases, as the distance increases (meaning the angle Θ is getting smaller and smaller), the reflection coefficient approaches -1. In fact another assumption made by Bullington was that r=-1. He was justified in that he was talking about antennas separated by many miles. But this doesn't work for the average microwave communication system used in the home or office.

The significance of r is that if it is not exactly -1 (or +1) the cancellation at the receiver will be incomplete and the fades and nulls will not be as deep. In most cases this is a good thing.

Now let's look at the case for sea water where $\epsilon = 80$ and $\sigma = 4$. First for Horizontal Polarization, then for Vertical Polarization.



Horizontal Polarization
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
Reflection Coefficien
0
5
10
15
20
25
30
Distance−−Meters


Vertical Polarization
0.8
0.6
0.4
0.2
0.0
−0.2
Reflection Coefficient
0
5
10
15
20
25
30
Distance−−Meters

Electric Field at Receiving Antenna.

The received signal at the receiving antenna is of an amplitude and phase equal to:

$$E = E_0(1 + r\varepsilon^{j\Delta} + \text{ some ground wave terms})$$

Or

$$\frac{E}{E_0} = (1 + r\varepsilon^{j\Delta} + \text{ some ground wave terms})$$

Where

E is the received field intensity

E_0 is the field intensity at the transmitting antenna

R is the reflection coefficient already derived.

Δ = Angdiff already derived.

At microwave frequencies, we can neglect the ground wave terms giving:

$$\frac{E}{E_0} = (1 + r\varepsilon^{j\Delta})$$

What does $e^{i\Delta}$ mean? Briefly it is a way to easily solve differential equations. Let me explain. To begin mathematically

$$e^{j\Delta} = Cos\left(\Delta\right) + jSin\left(\Delta\right)$$

If a linear system is driven by a sin or cos forcing function, the result must be a sin or cos function with a different amplitude and phase. If it is driven with both functions the result will be the superposition of the two. Now suppose that it was driven with the function cos +jsin. The result will be the superposition of the two. If at that time we want to know what the result of the cos forcing function was, simply take the real part of the result.

Since it is frequently easier to use exponentials to the solution of differential equations, it is simpler to drive with the function $e^{j\Delta}$ and then take the real part of the result to get the output due to the cos(Δ) portion of the input forcing function.

The system we are trying to model is a linear system so superposition should apply here as well.

In this function, r is possibly imaginary, so it makes since to keep it in exponential form. And the bottom line is that the forcing function we are interested in is a cos. One antenna is radiating a cos wave at some frequency. It ground bounces and heads off to the other antenna where its phase and amplitude is changed. Assuming we wanted to see the effect of the Cos forcing function, take the real part of the result and that will be the result of the forcing function.

Thus the function below becomes:

$$\frac{E}{E_0} = \left[1 + re^{j\Delta}\right] = \left[1 + \mathrm{Re}\left[re^{j\Delta}\right]\right]$$

Assuming for the moment that r is real, this becomes equal to

$$\frac{E}{E_0} = \left(1 + \mathrm{Re}\left[re^{j\Delta}\right]\right) = \left(1 + rCos\left(\Delta\right)\right)$$

Now the following trig identity is true

$$Cos\left(2t\right) = 1 - 2S\,\mathrm{in}\left(t\right)^2$$

Let $2t = \Delta$ and substituting E/E0 one obtains my earlier result

$$\frac{E}{E_0} = \left(1 + rCos\left(\Delta\right)\right) = 1 + r\left[1 - 2\left[Sin\left(\frac{\Delta}{2}\right)\right]^2\right]$$

For low angles, r=-1 and this equation becomes

$$\frac{E}{E_0} = 2\left[Sin\left(\frac{\Delta}{2}\right)\right]^2$$

Bullington further approximated this to be

$$\frac{E}{E_0} = 2\left[Sin\left(\frac{\Delta}{2}\right)\right] \approx \Delta$$

Again, he was dealing with very small angles and for small angles this assumption is justified.

Furthermore, he earlier made the assumption that the angle d is equal to:

$$\Delta = \frac{4\pi f h1 h2}{(c)(d)}$$

Thus his result is:

$$\frac{E}{E_0} = \frac{4\pi f h1 h2}{(c)(d)}$$

The actual angle from previous work is

$$\Delta = \frac{2\pi f\left(\sqrt{(h1+h2)^2 + d^2} - \sqrt{(h1-h2)^2 + d^2}\right)}{c}$$

Equation for E/E$_0$

I will use the following equation throughout the rest of the paper

$$\frac{E}{E_0} = \left(1 + rCos(\Delta)\right) = \left(1 + rCos\left(\frac{2\pi f\left(\sqrt{(h1+h2)^2 + d^2} - \sqrt{(h1-h2)^2 + d^2}\right)}{c}\right)\right)$$

The short distance problem--ψ

For very short distances there is a problem. The energy for the direct path has much less attenuation due to path loss then the indirect path. Visualize a situation where two antennas are mounted 10 meters above the ground plane, but are only 1 meter apart. The direct path has only 1 meter to go while the indirect path has 20 meters to travel. The indirect path will arrive with much less power than the direct path due to simple path loss. To see how to correct for this we note that field strength at the receiving antenna E is

$$E = \frac{\sqrt{30 g1 P1}}{d} \text{ volts/meter}$$

Where g1 is the antenna gain of the transmitting antenna, P1 is the transmitter power and d is the distance between antennas.

Then the field strength for the direct path is:

$$E_{dlos} = \frac{\sqrt{30 g1 P1}}{dlos} \text{ volts/meter}$$

And the field strength for the indirect path is:

$$E_{mp} = \frac{\sqrt{30g1P1}}{mp} \text{ volts/meter}$$

With this information, how might we modify the previous equation restated below:

$$\frac{E}{E_0} = \left(1 + rCos\left(\Delta\right)\right)$$

It should be noted that this effects only the amplitude not the phase. The phase was already taken care of with the term Δ.

We could modify both the 1 (the direct path) and the tem in from of the cos (the indirect path), but this would make it more complicated and screw things up later when we wanted to add in path loss analysis. It is simpler to normalize the terms to assume that the direct path is the reference and the indirect path gets "adjusted". We do this by the following:

$$\frac{E}{E_0} = \frac{\left(E_{dlos} + rE_{mp}Cos\left(\Delta\right)\right)}{E_{dlos}} = 1 + r\frac{E_{mp}}{E_{dlos}}Cos\left(\Delta\right)$$

Now

$$\frac{E_{mp}}{E_{dlos}} = \frac{\frac{\sqrt{30g1P1}}{mp}}{\frac{\sqrt{30g1P1}}{dlos}} = \frac{dlos}{mp}$$

From previous work

$$\mathrm{mp} = \mathrm{L1} + \mathrm{L2} = \sqrt{d^2 + (\mathrm{h1} + \mathrm{h2})^2}$$

$$\mathrm{dlos} = \sqrt{(\mathrm{h1} - \mathrm{h2})^2 + d^2}$$

Thus

$$\frac{dlos}{mp} = \frac{\sqrt{\left(h1 - h2\right)^2 + d^2}}{\sqrt{\left(h1 + h2\right)^2 + d^2}}$$

I am going to define this ratio as ψ

Therefore

$$\psi = \frac{dlos}{mp} = \frac{\sqrt{(h1-h2)^2 + d^2}}{\sqrt{(h1+h2)^2 + d^2}}$$

And

$$\frac{E}{E_0} = \frac{\left(E_{dlos} + rE_{mp}Cos(\Delta)\right)}{E_{dlos}} = 1 + r\frac{E_{mp}}{E_{dlos}}Cos(\Delta) = 1 + r\psi Cos(\Delta)$$

Finally

$$\frac{E}{E_0} = \left(1 + r\psi Cos(\Delta)\right) = \left(1 + r\psi Cos\left(\frac{2\pi f\left(\sqrt{(h1+h2)^2 + d^2} - \sqrt{(h1-h2)^2 + d^2}\right)}{c}\right)\right)$$

With r equal to

$$r_h = \frac{h1 + h2 - \sqrt{d^2(\varepsilon - 1) + (h1+h2)^2 \varepsilon}}{h1 + h2 + \sqrt{d^2(\varepsilon - 1) + (h1+h2)^2 \varepsilon}} \qquad \text{Horizontal Polarization}$$

$$r_v = \frac{\varepsilon(h1 + h2) - \sqrt{d^2(\varepsilon - 1) + (h1+h2)^2 \varepsilon}}{\varepsilon(h1 + h2) + \sqrt{d^2(\varepsilon - 1) + (h1+h2)^2 \varepsilon}} \qquad \text{Vertical Polarization}$$

I recognize that this is a complicated equation with the new values for r and ψ, but it is what is needed to describe the problem at short range microwave frequencies. Fortunately we now have programs like Mathematica that allow us to easily solve these equations.

Path Loss

The ratio of power received to power transmitted is:

$$\frac{\mathrm{P2}}{\mathrm{P1}} = (\frac{\lambda}{4\pi d})^2 \mathrm{g1g2}(\frac{E}{\mathrm{E0}})^2$$

Where g_1 is the antenna gain of antenna 1 and g2 is the antenna gain of antenna 2. P_2 is the received power and P_1 is the transmitted power. Expressed in dB, the received power will be:

$$P_{2dBm} = 20Log_{10}\left[\frac{\lambda}{4\pi d}\right] + 10Log_{10}\left[g1\right] + 10Log_{10}\left[g2\right] + 20Log_{10}\left[\frac{E}{E_0}\right] + P_{1dBm}$$

The first term is a gain term (which will be negative) and when flipped as below is called the path loss or PL

$$PL = 20Log_{10}\left(\frac{4\pi d}{\lambda}\right)$$

If there were no indirect path signal, the direct path signal would be straightforward and equal to:

For the direct path signal

$$P_{dp} = 20Log_{10}\left[\frac{\lambda}{4\pi dlos}\right] + 10Log_{10}\left[g1\right] + 10Log_{10}\left[g2\right] + P_{1dBm}$$

However the indirect path signal interferes with the direct path signal causing nulls and fades as described earlier by the equation

$$20Log_{10}\left[\frac{E}{E_0}\right] = 20Log_{10}\left[\left(1 + r\psi Cos\left(\Delta\right)\right) = \left(1 + r\psi Cos\left(\frac{2\pi f\left(\sqrt{\left(h1+h2\right)^2 + d^2} - \sqrt{\left(h1-h2\right)^2 + d^2}\right)}{c}\right)\right)\right]$$

If we add this equation to the direct path power, we have the complete received power signal at the receiving antenna described in the following equation.

$$P_{2dBm} = 20Log_{10}\left[\frac{\lambda}{4\pi dlos}\right] + 10Log_{10}\left[g1\right] + 10Log_{10}\left[g2\right] + 20Log_{10}\left[\frac{E}{E_0}\right] + P_{1dBm}$$

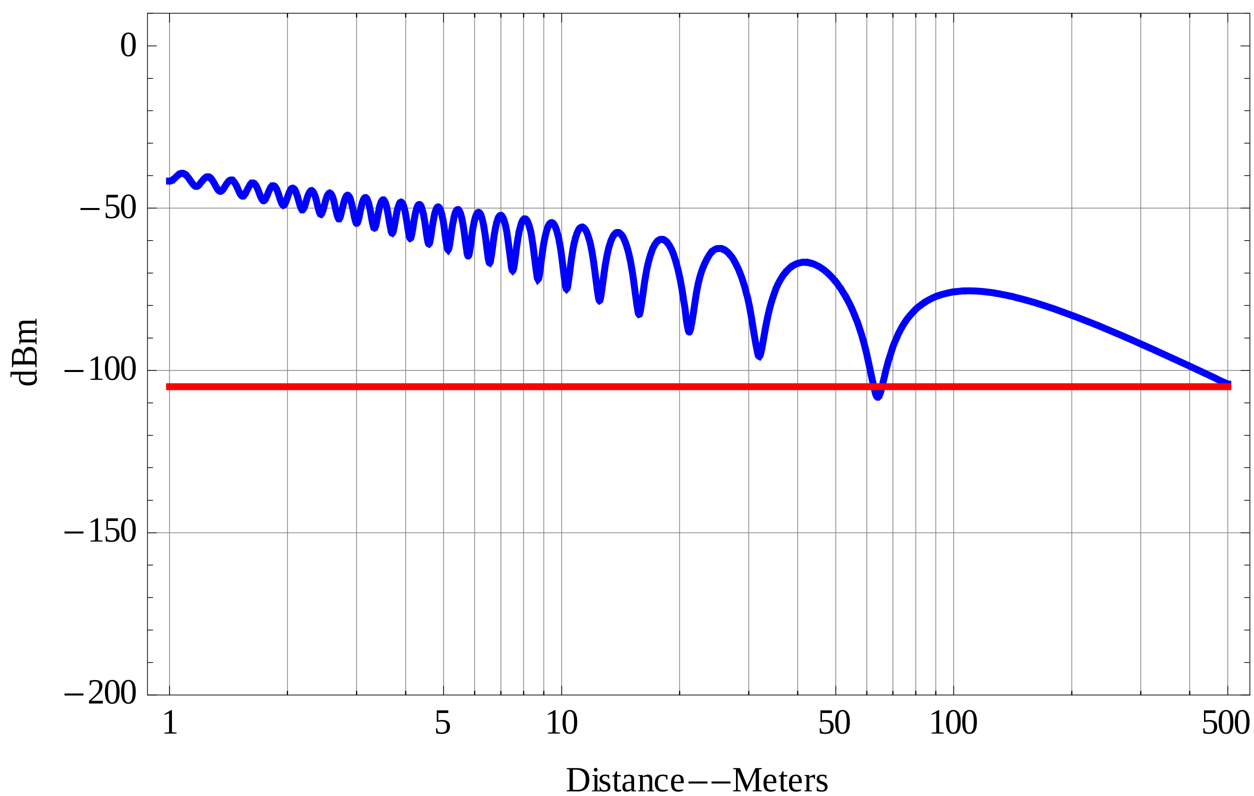
In this equation, the term $20Log_{10}\left[\frac{E}{E_0}\right]$ is automatically a multipath signal and has already had the multipath distance taken into account.

Examples

Let's now look at some examples: I have plotted these functions in Mathematica. The results are pasted below. Frequency is 2400 MHZ in all cases

In the first graph,

Transmit power	0 dBm
Polarization	Horizontal
Dielectric Constant	30
H1	2 meters
H2	2 meters
Blue trace	P_{2dBm}
Red trace	Receiver Sensitivity

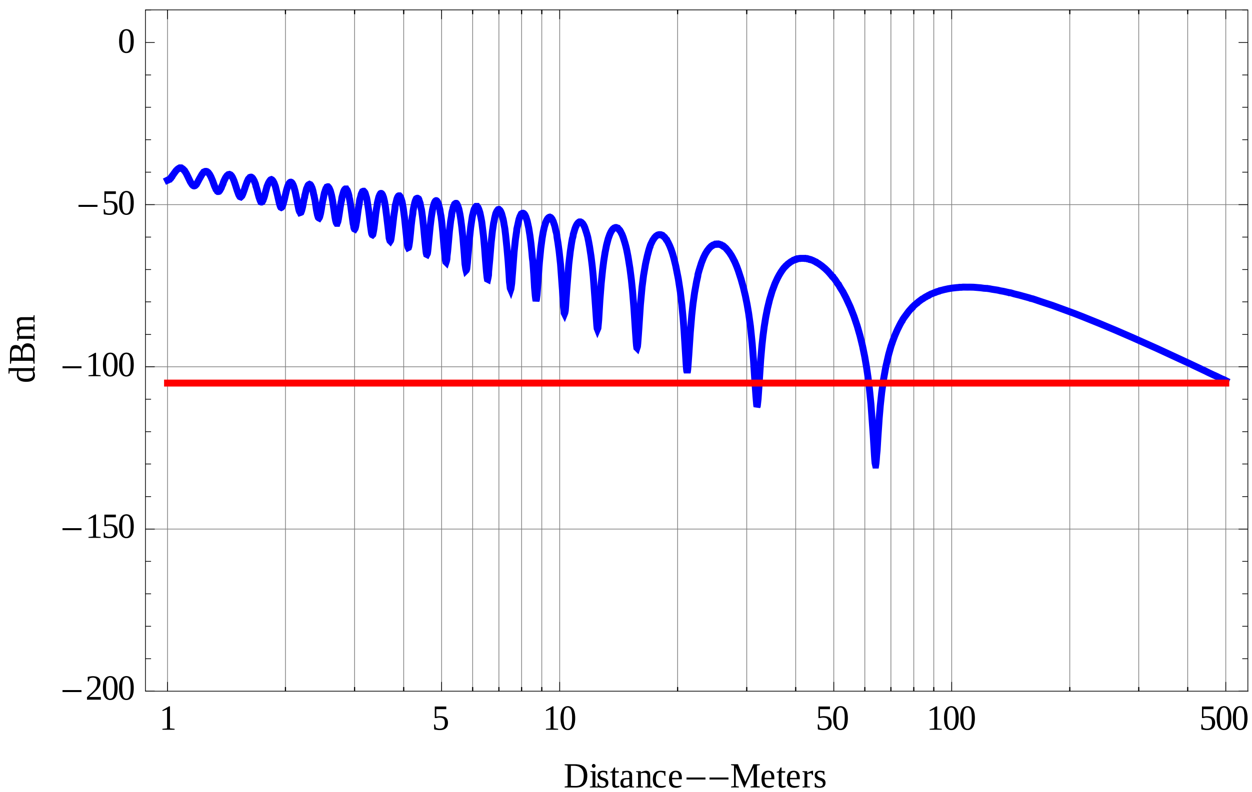


Observe how the nulls get deeper as the distance increases. This is because ψ is approaching unity as d increases and the cancellation resulting from the indirect signal and the direct path signal is becoming more complete.

If the receiver sensitivity (shown by the red trace) is above the blue trace, a loss of signal would occur. In this graph, such a condition occurs at about 62 meters and from 500 meters on.

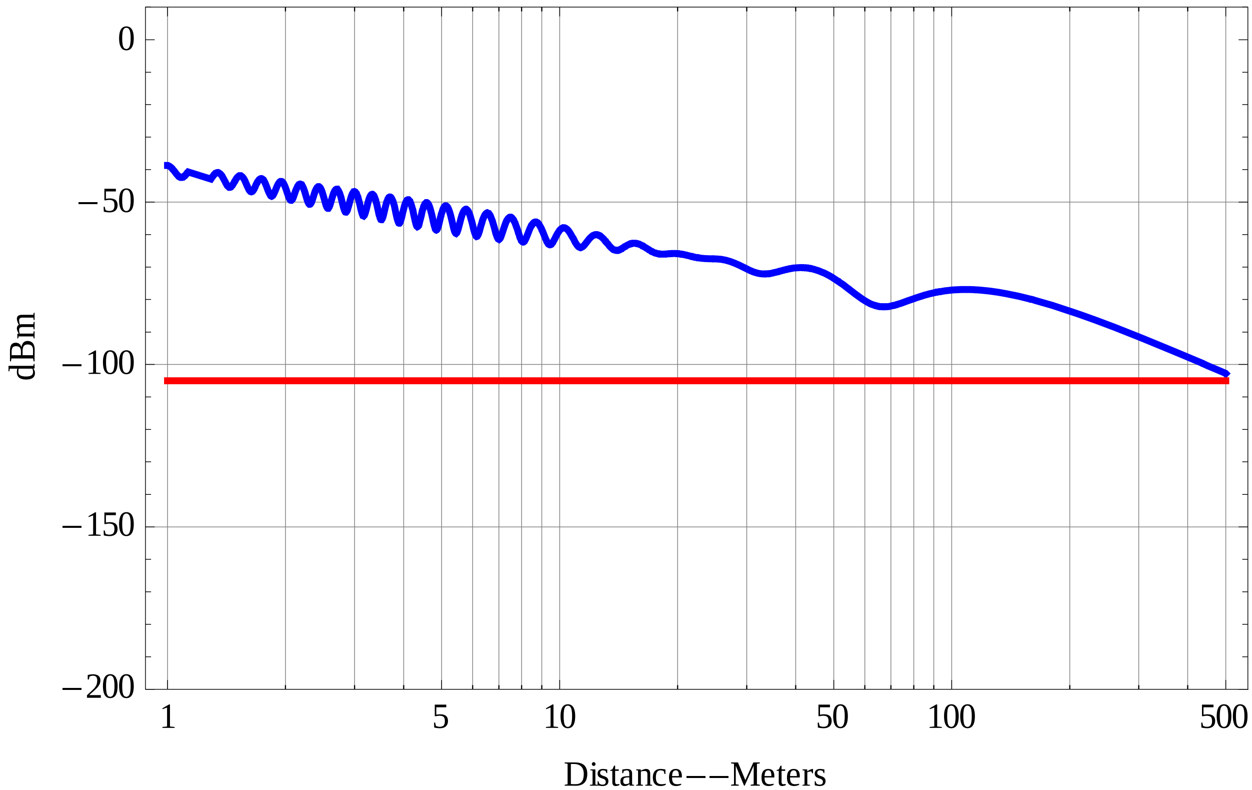
The next graph shows the same information but with the assumption that the reflection coefficient is now r= -1 for all distances. (we replace the complicated equation for r with -1).

Observe how the nulls are deeper than case where we used the actual value for r. As d gets larger the effect is mitigated by the fact that r approaches -1 for small angles



r=-1

Now take a quick look for vertical polarization

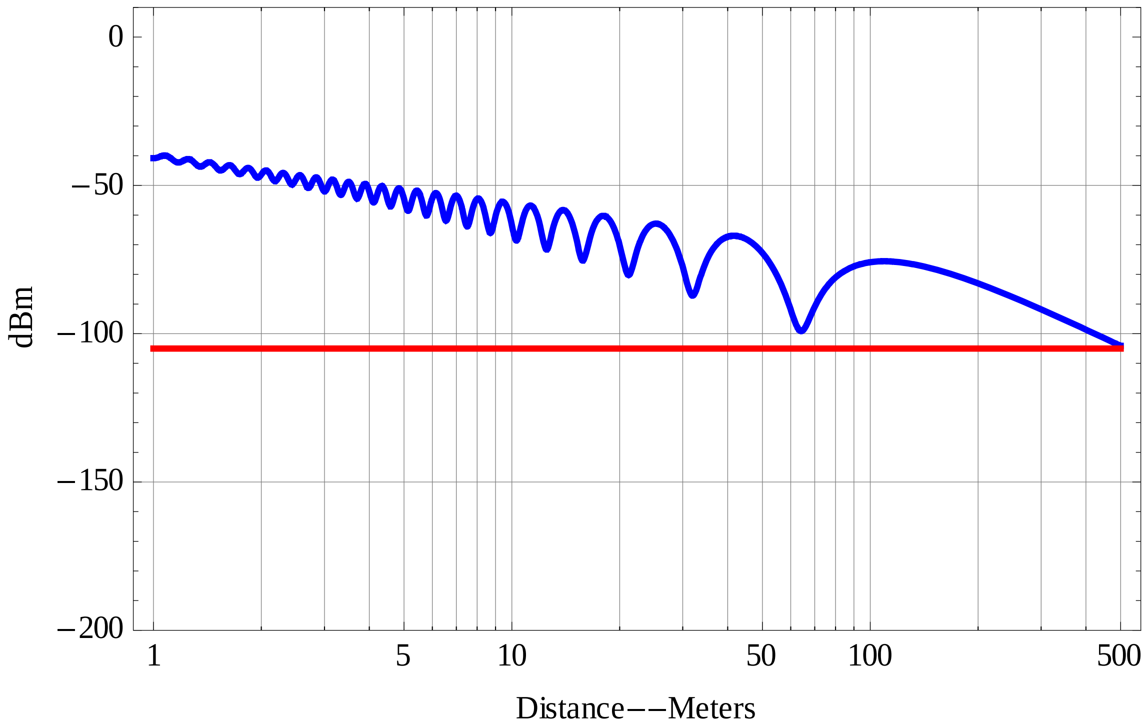


Vertical Polarization

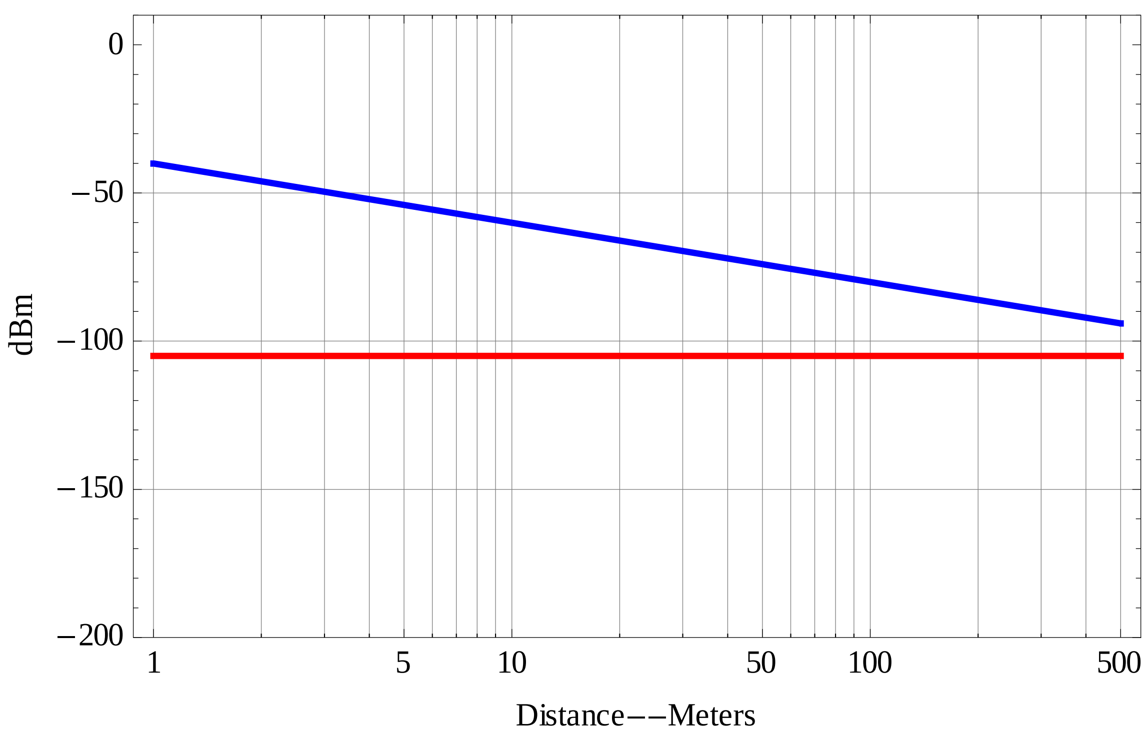
Vertical polarization results in the reflection coefficient starting positive and then going through zero on its way to -1 as the angle gets small (d gets large). Brewsters angle (the angle at which r =0, occurs at approximately 25 meters for this particular situation.

Now let's go back to horizontal polarization but change the dielectric constant to 4 (for poor earth ground).

Compare that against the first graph where the dielectric constant was 30. In this case the ground reflects much less energy so the nulls are less pronounced. In fact, if we set the dielectric constant to 1, the same as air, there will be no reflection and therefore no nulls at all. This is shown in the following graph.



Dielectric constant is 4

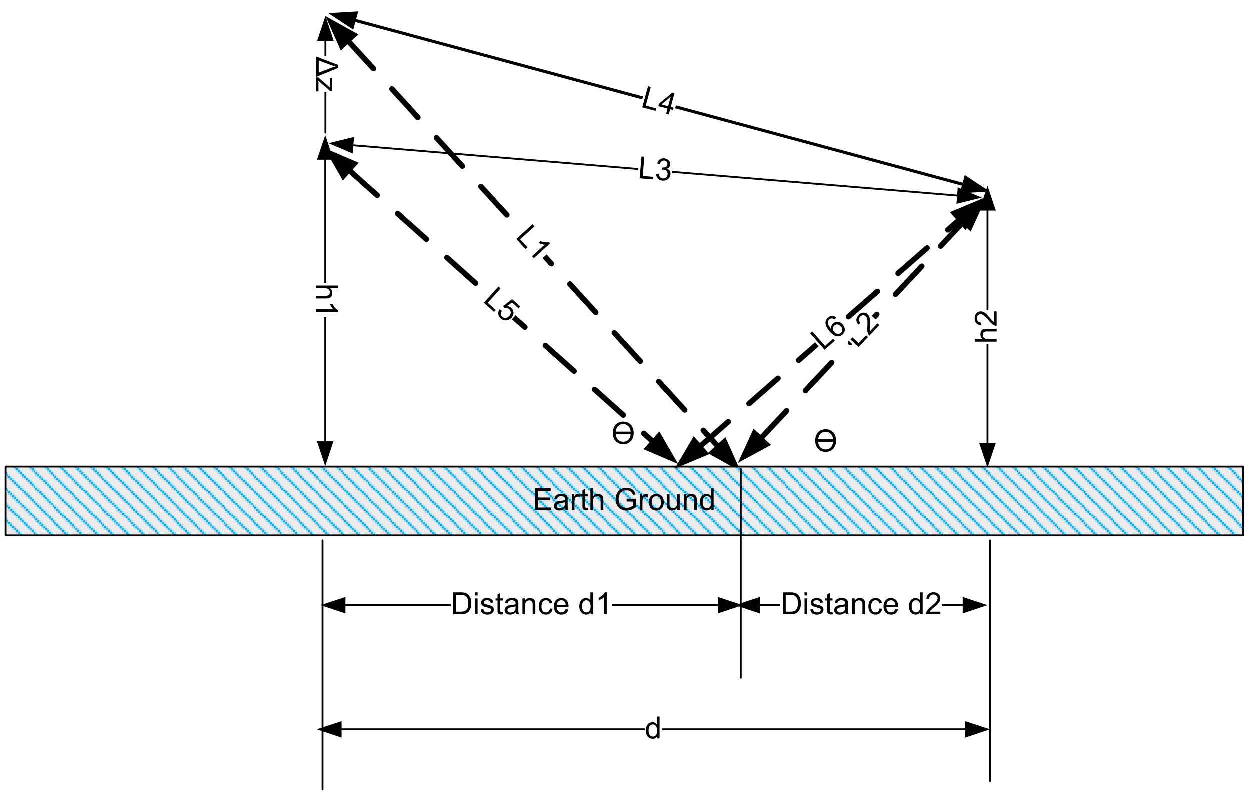


Dielectric constant is 1

It is interesting to note that the model can do more than model a ground reflection. It can model any reflection off of any surface. For example supposing a wall in a building is causing a multipath reflection. Simply input the two heights of the antennas as the distance from each antenna to the wall. And if the antennas are horizontally polarized with respect to earth ground, then one would have to flip the polarization as a wall would represent vertical polarization in this instance.

Diversity Antennas

To improve multipath rejection a technique called diversity antennas is employed. In this case, a third antenna is mounted on the receiver a short distance from the receiver's first antenna. By spacing them apart, one of the antennas is likely not in a null. The electronics in the receiver looks at the signal strength of the received signal from both antennas and selects the antenna with the stronger signal.



Antenna system with Diversity Antennas.

Observe that in this antenna system, the distances L3 will have changed to L4. In addition, L5 will have changed to L1 and L6 will have changed to L2. And of course, d1 and d2 will have changed as will the angle ϴ.

These changes are all due to antenna 1 changing its height from h1 to h1+ΔZ. Fortunately all of our previous equations hold. All we have to do is change in the equations the height of antenna 1 from h1 to h1 + ΔZ. We already have the signal strength at the antenna 1 height of h1. Now let's look at what it would be at antenna height of h1+ ΔZ.

$$\frac{E}{E_0}\left(h1+\Delta Z\right)=\left(1+r\psi Cos\left(\Delta\right)\right)=\left(1+r\psi Cos\left(\frac{2\pi f\left(\sqrt{\left(h1+\Delta Z+h2\right)^2+d^2}-\sqrt{\left(h1+\Delta Z-h2\right)^2+d^2}\right)}{c}\right)\right)$$

$$r_h\left(h1+\Delta Z\right)=\frac{h1+\Delta Z+h2-\sqrt{d^2\left(\varepsilon-1\right)+\left(h1+\Delta Z+h2\right)^2\varepsilon}}{h1+\Delta Z+h2+\sqrt{d^2\left(\varepsilon-1\right)+\left(h1+\Delta Z+h2\right)^2\varepsilon}}\qquad \text{Horizontal Polarization}$$

$$r_v\left(h1+\Delta Z\right)=\frac{\varepsilon\left(h1+\Delta Z+h2\right)-\sqrt{d^2\left(\varepsilon-1\right)+\left(h1+\Delta Z+h2\right)^2\varepsilon}}{\varepsilon\left(h1+\Delta Z+h2\right)+\sqrt{d^2\left(\varepsilon-1\right)+\left(h1+\Delta Z+h2\right)^2\varepsilon}}\qquad \text{Vertical Polarization}$$

$$\psi\left(h1+\Delta Z\right)=\frac{dlos}{mp}=\frac{\sqrt{\left(h1+\Delta Z-h2\right)^2+d^2}}{\sqrt{\left(h1+\Delta Z+h2\right)^2+d^2}}$$

$$\mathrm{dlos}(\mathrm{h1}+\Delta\mathrm{Z})=\sqrt{(\mathrm{h1}+\Delta\mathrm{Z}-\mathrm{h2})^2+d^2}$$

The complete received power at the antenna height is now equal to :

$$P_{2dBm}\left(h1+\Delta Z\right)=20Log_{10}\left[\frac{\lambda}{4\pi dlos\left(h1+\Delta Z\right)}\right]+10Log_{10}\left[g1\right]+10Log_{10}\left[g2\right]+20Log_{10}\left[\frac{E}{E_0}\left(h1+\Delta Z\right)\right]+P_{1dBm}$$

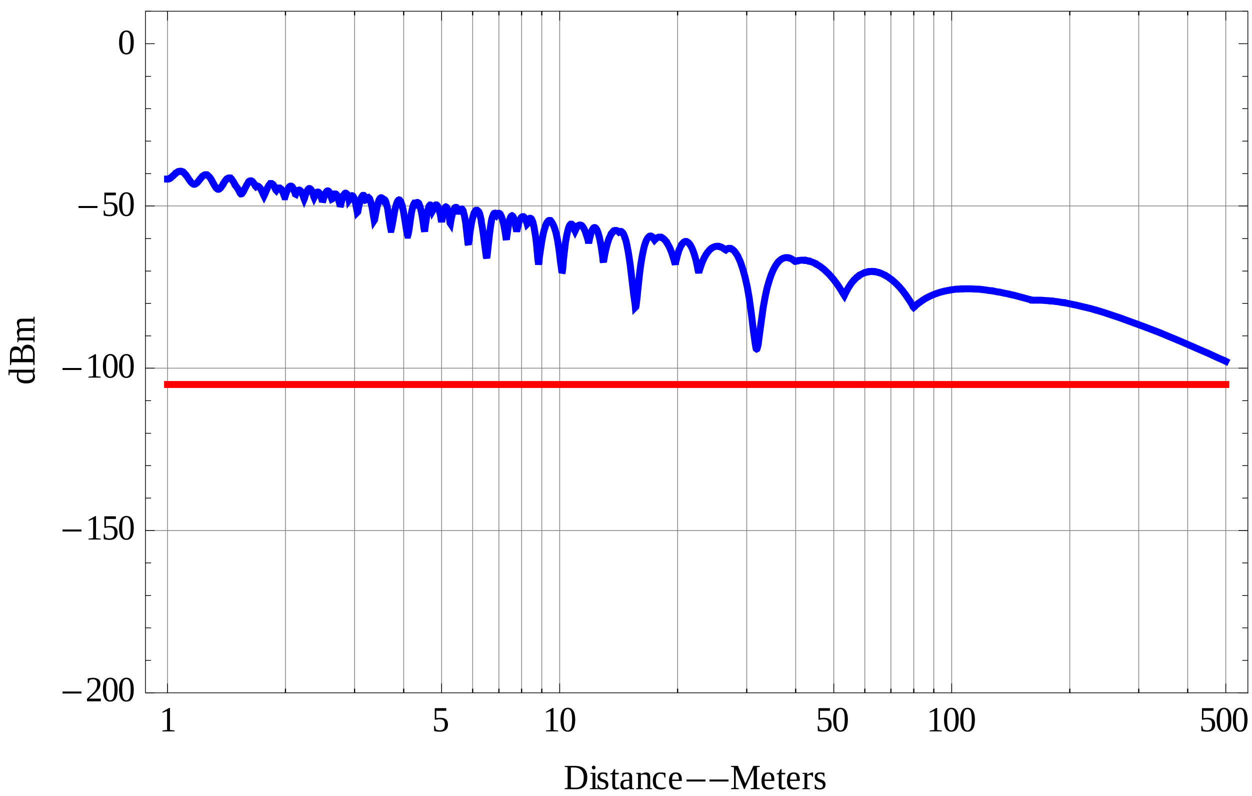
The electronics in the receiver would look at the signal P_{2dBm} and P_{2dBm}(h1+ΔZ) and select the larger of the two.

Examples

Let's look at some examples. We will begin with the same example as before, but now we will let ΔZ = 1 wavelength.

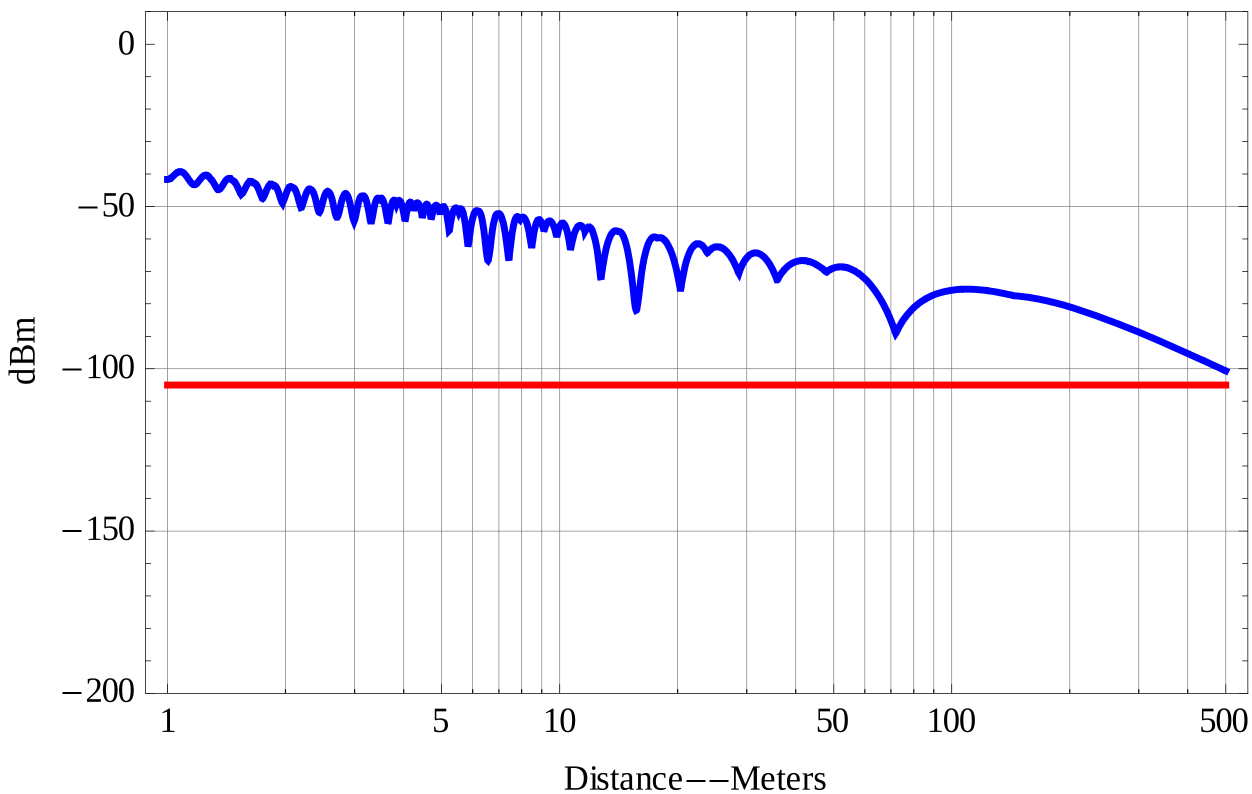
Transmit power	0 dBm
Polarization	Horizontal
Dielectric Constant	30
H1	2 meters
H2	2 meters
Blue trace	P_{2dBm}
Red trace	Receiver Sensitivity

The graph is shown below.



ΔZ = 1

Observe how the nulls have been attenuated. This is because when one antenna is experiencing a null, the other one probably isn't. So one of the two receive signals will have a stronger signal.

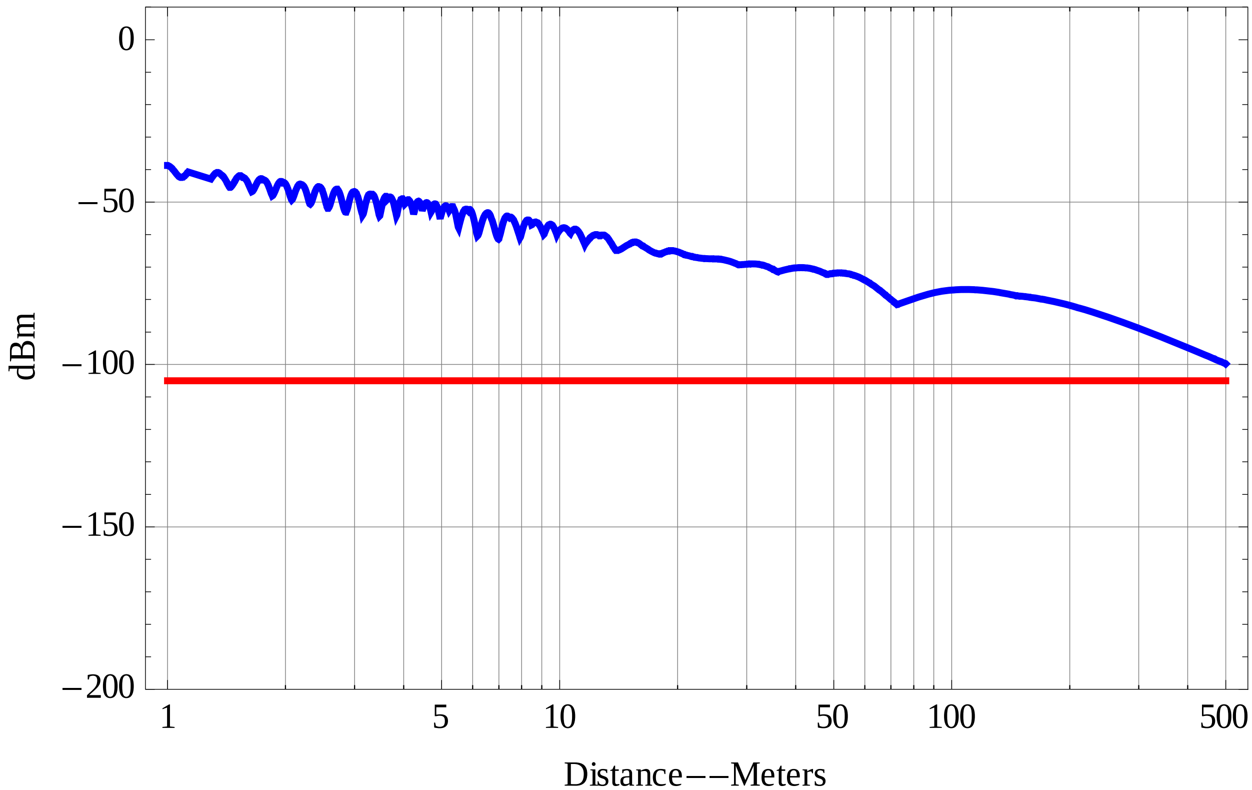


ΔZ = 0.5

Same as previous except ΔZ is now a half wavelength. Notice how the nulls are now at least 20 db from the red line (indicating maximum receiver sensitivity) making this much more noise immune than without the diversity antennas.

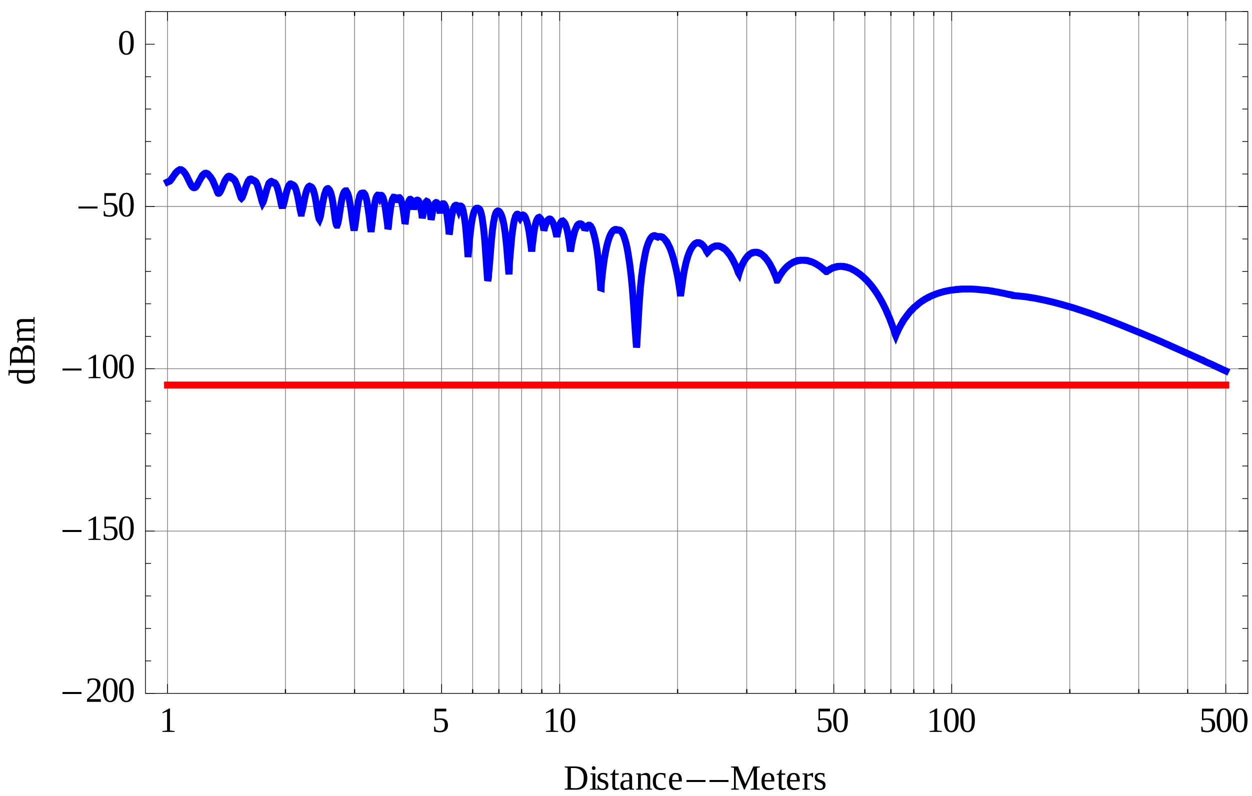
Transmit power	0 dBm
Polarization	Vertical
Dielectric Constant	30
H1	2 meters
H2	2 meters
Blue trace	P_{2dBm}
Red trace	Receiver Sensitivity

Now look at vertical polarization and a ΔZ of a half wavelength.



Vertical Polarization with ΔZ = 0.5

Finally look at the case where the reflection coefficient is -1 and the ΔZ is a half wavelength.



Reflection Coefficient is -1 and ΔZ = 0.5

This is a much more useable signal than with ΔZ zero. With ΔZ zero, several of the nulls crossed the red line.